Patents, Litigation Strategy and Antitrust In Innovative Industries

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March 2019

Abstract

In a patent infringement suit, the alleged infringer wins with a ruling of either patent invalidity or non-infringement. It is ambiguous which of these outcomes is preferred by the alleged infringer. Invalidity may increase current-period competition, but simultaneously removes constraints to successful future innovation. The choice of whether to vigorously pursue patent invalidation may also affect incentives to innovate. We adapt the "innovative industries" model of Segal and Whinston (2007) to study patent litigation strategy and rates of innovation. We show that a legal regime where infringement is considered first (and validity second) maximizes incentives to innovate. But if the future blocking effect of patent validity is strong, the alleged infringer may prefer to litigate validity first to maximize the likelihood of invalidity. This litigation strategy effect may reduce levels of innovation. Antitrust policy should seek to attenuate this effect, and may do so by reducing the advantage to incumbency.

JEL Classification: *K2, L4, O3.*
Keywords: antitrust, invalidity, litigation, non-infringement, patents.

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1. Introduction

This paper studies characteristics and implications of patent invalidity and patent non-infringement. Since the early 1990s, there has been a marked increase in the importance of pretrial litigation maneuvers for outcomes in patent litigaiton.[1] Some of the more common maneuvers (e.g. requests for a particular claim construction in a "Markman" hearing, or a motion for summary judgment) give alleged infringers significant ability to influence the likelihood of invalidity and non-infringement rulings (Ford 2013). Firms have also asked courts to vacate rulings as part of patent litigation settlements, and courts have often granted these requests (Bock 2013).[2]

Unfortunately, the incentives to alter probabilities of invalidity and non-infringement, and the effects on innovation of doing so, are not well understood. Many researchers have argued that the incentives to invalidate patents are too low, as alleged infringers may not pursue invalidity due to the relatively high burden of proof (Ford 2013) and may not internalize the benefits of patent invalidity accruing to other firms and consumers (Farrell and Merges 2004). But previous consideration of these phenomena has been largely informal, and no research has used a dynamic model to study the effects on innovation of changes to *expected* rates of invalidity and non-infringement. We take steps to fill both gaps.

We introduce a model that delineates current and future effects, on a patentee and a single alleged infringer, of replacing patent invalidity with non-infringement. The model assumes that all non-patentee firms (including a group of other competitors) may use the patented technology in current and future periods under invalidity. Under non-infringement, in contrast, there are some restrictions. In the current period, the alleged infringer may continue using its own (non-infringing) technology, but other firms may not imitate the patented technology. And in the future, all non-patentee firms may not use the patented technology in pursuing innovation that determines which firm has dominant technology at

[1] Henry and Turner (2016) include data showing that among all patent cases from 1976-1990 in the United States that reach a decision about infringement and/or validity, about 56% of cases are decided by a trial and 44% are decided by a pre-trial motion. These fractions are the same in 1976-80, 1981-85 and 1986-90. After 1990, the fraction changes. During 1991-95, about 50% of cases are decided by a trial. During 1996-2000, about 41% of cases are decided by a trial.

[2] Among 79 patent cases with settlement-related motions for vacatur filed during 2006-2011, Bock (2013) finds the court grants the motion in 62 of them.

the start of a second period of competition.

This model highlights when and how a patentee and alleged infringer may have different preferences over invalidity and non-infringement. The patentee obviously prefers non-infringement, which limits competition in the current period and for future innovation. But for the alleged infringer, there is a tradeoff. In the current period, it benefits more from patent non-infringement, as the patent prevents other firms from imitating technology; effectively, the alleged infringer is able to "slipstream" behind its rival's (valid) patent and face less competition. For the future, it benefits more from patent invalidity, as a valid patent "blocks" it from using its rival's technology in cumulative innovation.

We then embed slipstreaming and blocking effects (in reduced form) in a dynamic model and analyze equilibrium innovation and legal strategy. In particular, we adapt the "innovative industries" model of Segal and Whinston (2007), where entrants innovate and patent in attempting to replace current incumbents and gain dominant market positions. Our adaptations specify current-period payoffs, and the probability that the entrant replaces the incumbent as the future dominant firm, as functions of the outcome of uncertain patent litigation.

We find that a change in legal regime that raises the likelihood of non-infringement, while reducing the likelihood of invalidity, promotes innovation. Specifically, equilibrium innovation is higher under the "infringement first" (IF) regime (which always considers infringement) than under the "validity first" (VF) regime (which always considers validity). The expected reward for an innovator is higher under IF for two reasons. Expected current-period payoffs are higher for both firms, because a valid-but-not-infringed patent deters imitation from fringe firms while an invalid patent does not. And the entrant innovator is more likely to replace the incumbent as the dominant firm, because of the future blocking effect of patent validity.

We then allow the legal regime (IF or VF) to be determined endogenously in the model. Consistent with the seniority of validity in patent law, we specify that VF obtains if either firm prefers it. Because the entrant patentee is better off in the current-period and future under non-infringement, it always prefers IF. But due to the tradeoff between the slipstreaming and blocking effects, the incumbent alleged infringer may prefer either IF or VF. The alleged infringer's preferences are therefore decisive.

In analyzing equilibrium regime choice and innovation, there are multiple possibilities. But two stark cases sharply highlight the key tradeoffs. On one hand, if there is a strictly positive slipstreaming effect but zero blocking effect, then IF and a relatively high rate of innovation obtain in equilibrium. Intuitively, this is a "live and let live" equilibrium. It will tend to obtain in an industry with two leading firms that do not compete fiercely on price and have highly differentiated technologies (so that it is not difficult to invent around patents).

On the other hand, if the slipstreaming effect is zero but there is a strictly positive blocking effect, then VF and a relatively low rate of innovation obtain in equilibrium. Intuitively, this is a "scorched earth" equilibrium. It will tend to obtain in an industry where firms compete fiercely on price (e.g., à la Bertrand) and have homogeneous technologies. The latter case yields a "litigation strategy effect"—the option to choose the legal regime yields a rate of innovation that is lower than it would be under mandated IF.

With strictly positive slipstreaming and blocking effects, equilibrium depends upon the relative sizes of the effects. Interestingly, multiple equilibria—"live and let live" and "scorched earth"—are possible in the model. Intuitively, higher innovation decreases the incumbency advantage—the additional payoff from being the incumbent instead of the entrant—and this reduces the alleged infringer's expected loss from the blocking effect, reinforcing its incentive to choose IF. Similarly, lower innovation under VF increases the incumbency advantage, increasing the alleged infringer's expected loss from the blocking effect and reinforcing its incentive to choose VF.

The model permits some analysis of comparative static effects of various parameters on the likelihood that there is an equilibrium where IF is chosen. Any increase in the expected replacement probability increases the incentive to innovate and lowers the incumbency advantage. In general, this increases the likelihood that there is an equilibrium where IF is always chosen and attenuates the litigation strategy effect. However, the caveat is when there is an increase in the probability that the entrant replaces the incumbent after a "not infringed" decision. This decreases the incumbency advantage, but simultaneously increases the incumbent's likelihood of retaining its incumbency under VF relative to IF. The former increases the likelihood of an IF equilibrium, while the latter decreases the likelihood of an IF equilibrium. Thus, the total effect is ambiguous.

Antitrust enforcement should seek both to promote innovation incentives and to attenuate the litigation strategy effect. In studying policy towards unilateral actions that dominant incumbents might take, we follow Segal and Whinston (2007) in assuming that enforcement raises the expected current-period profit for the entrant. As in their model, there are clear circumstances (e.g., prohibiting predatory pricing) where antitrust enforcement raises the joint current-period profit of the firms and thereby enhances incentives to innovate, under both the IF and VF regimes. More importantly, we find that whenever the effect of antitrust enforcement is higher in magnitude for the entrant's current-period payoff than for the incumbent's current-period payoff, the incumbency advantage falls. This attenuates the litigation strategy effect and indirectly yields even higher innovation incentives.

We also consider policy against multilateral agreements. When multiple equilibria may obtain, firms might try to coordinate to achieve a particular equilibrium. In this case, studying attempts to use antitrust policy (to attenuate the litigation strategy effect) sharply highlights the tension between antitrust enforcement and intellectual property. Consider an agreement among rivals (or an industry norm) to always choose the IF regime and eliminate the possibility of a VF equilibrium. Antitrust authorities are likely to take a dim view of such agreements. Suppose, alternatively, that two rivals agree not to file petitions for inter partes review with the Patent Trials and Appeals Board (PTAB), but otherwise compete the same (in product markets and in litigation). This has the same basic effect as an agreement to always choose IF. But in settings captured by our model, this sort of shift actually would attenuate the litigation strategy effect and weakly raise innovation incentives.

This paper formalizes the effects of patent non-infringement versus invalidity in a stylized model. Our slipstreaming effect encompasses ideas from Farrell and Merges (2004) and Ford (2013), who offer detailed discussions of the external effects of patent validity absent infringement (which we discuss below). Our blocking effect on future innovation captures the well-understood idea that patents may stifle future innovation by blocking access to a technology (e.g., Murray and Stern 2007; Williams 2013; Galasso and Schankerman 2015).

Our analysis of the relationship between characteristics of patent litigation and innovation builds directly off of the Segal and Whinston (2007) framework.[3] In furthering their effort to provide understanding of the effects of static market power on dynamic incentives to innovate, our paper relates to the literature on the effects of policies that alter the market power of technology leaders when the primary nexus of competition is "for the market" instead on "in the market" (Evans and Schmalensee 2002).[4] Both the slipstreaming and blocking effects are likely to be present, in some form, in industries where direct competitors also attempt to produce drastic innovations.[5]

We also complement recent work by Gans (2010) and Gans and Persson (2013), who study similar issues in an environment where an innovating entrant and a (patent-holding) incumbent may cooperate in commercializing new technology. They add bargaining over a license to the basic Segal and Whinston (2007) framework and study incentives to innovate and the effect of antitrust enforcement. Cooperative bargaining alters the effects of antitrust enforcement on innovation incentives in important ways. For example, Segal and Whinston (2007) show that antitrust enforcement that raises the incumbent's payoff under competition increases the innovation reward (by raising the value of incumbency). In contrast, Gans (2010) and Gans and Persson (2013) show that antitrust enforcement that *lowers* the incumbent's payoff under competition increases the innovation reward. Intuitively, the entrant shares the bargaining surplus from cooperating instead of competing, and this is higher when the incumbent's competitive payoff is lower.

Our work also sheds light on the literature analyzing how patent lawsuits may help to

[3]Like Segal and Whinston, we build upon the classic literature on incentives to innovate (e.g., Arrow 1962) as well as the more recent "patent race" literature (e.g., Loury 1979; Lee and Wilde 1980), where R&D intensity is driven by the reward that an innovator may receive from achieving a dominant market position. And in terms of modeling, we also more closely adhere to the growth literature (e.g., Grossman and Helpman 1991; Aghion and Howitt 1992; Aghion et al. 1992), where there is an infinite horizon and a sequence of innovations.

[4]Evans and Schmalensee (2002) focus on antitrust enforcement, which is a secondary focus of our analysis.

[5]Industries with such competition have become increasingly common since the emergence of the so-called "new economy" in the 1990s, and patent litigation between market leaders has played a major role in new-economy industries. For example, Apple and Samsung contested patent litigation from 2011-18 as part of the "smartphone patent wars." This drew significant attention from economists, legal scholars and the financial media Economist (2010).

sort out bad patents. The problem of bad patents, i.e., patents that should not have been issued, evolves from inadequate resources at the patent offices and incentives faced by the examiners (Farrell and Merges, 2004). Against the background that patents can be a burden for future innovation by blocking access to a technology (Murray and Stern, 2007; Williams, 2013; Galasso and Schankerman, 2015), the existence of invalid patents may not only be detrimental to firm success but may also be factor slowing down technological progress and economic growth.

A number of authors (e.g., Lemley, 2001) have argued that the issuance of invalid patents by the patent offices is not necessarily inefficient because in reality only few patents really matter. Hence, it may be more cost effective to raise the question of validity of those patents in a courtroom rather than improving the application procedure for all patents.[6] However, Farrell and Merges (2004) challenge this view on the effectiveness of patent litigation to sort out invalid patents. They argue that beside a lack of resources of a challenger and the risk of losing, alleged infringer lack the incentives to challenge a patent. The latter argument follows from the public goods nature of invalidating a patent. If a patent is invalidated, it will not only benefit the challenger but also other competitors in the industry. Ford (2013) focuses on the procedural details of an alleged infringer's defense strategy, and compares a non-infringement strategy with an invalidity strategy. He first argues that it is difficult to pursue both strategies simultaneously[7] He also discusses advantages and disadvantages of non-infringement, and our slipstreaming and blocking effects reflect similar forces (p. 112). If the non-infringement strategy then dominates the invalidity strategy, litigation will not be able to sort out invalid patents effectively. Ford (2013) argues that this is indeed true because of the high standard to prove invalidity, an informational disadvantage on the matter of validity of the alleged infringer, and because of asymmetries of the outcome. He proposes that the problem of interdependence of defense strategies can be solved with a bifurcated system, i.e., a system where the matters of infringement and invalidity are handled separately. Whereas such a system exists in China and Germany, most other jurisdiction, including the

[6]Frakes and Wasserman (Forthcoming) study newer data and instead conclude that increasing examiner time would enhance efficiency.

[7]Ford (2013) argues that the success probabilities of the two strategies are not independent. Bringing forward an argument for invalidity may at the same time serve as an argument for infringement.

US, do not separate them (Cremers et al, 2016, 2017). However, a bifurcated system would not heal the potential dominance of the non-infringement strategy.

2. The Slipstreaming and Blocking Effects

This section motivates the slipstreaming and blocking effects. We discuss empirical phenomena that highlight situations when these effects have emerged. We also analyze a simple model of innovation and imperfect competition, which shows how/when the effects emerge and how they affect firm preferences over non-infringement versus invalidity.

2.1. Slipstreaming

Slipstreaming refers to the practice of a cyclist riding directly behind another, thereby reducing wind resistance and saving energy. When a rival's patent's validity is maintained, an allegedly infringing firm may similarly gain via reduced competition from other firms, who fail to enter and/or imitate the patentee's technology. We start with an example from the US pharmaceutical market.

In 1992, Imperial Chemical Industries (ICI) lost a ruling in a Paragraph IV patent infringement suit over its drug Tamoxifen [*Imperial Chem. Indus., PLC v. Barr Labs., Inc.* 795 F. Supp. 619, 626-27 (S.D.N.Y. 1992)]. The court found that ICI had intentionally withheld information from the US Patent and Trademark Office about tests for safety and effectiveness, and ruled their patent invalid. ICI appealed the ruling but before the appellate court came to a conclusion the parties then entered a settlement agreement that asked the court to vacate the finding of patent invalidity, and required that Barr drop its attempt to enter as a generic manufacturer. Brand manufacturer Zeneca (the patentee after ICI demerged its pharmaceuticals and agrochemicals businesses) also agreed to make significant "reverse" payments to Barr as part of this agreement (Carrier 2009, pp. 361-63).

Through this deal that restored patent validity, Zeneca and Barr sought to share in monopoly profits from Tamoxifen. For this to work, however, the patent needed to block subsequent entry by other rivals. To the great delight of the parties, Zeneca won three lawsuits at different courts with different judges against additional generic manufacturers attempting to challenge its patent and enter.[8] Barr benefited directly from the validity of Zeneca's patent.

Monopolization like this is an extreme case. More generally, patent non-infringement that constrains other rivals may improve the payoffs for the alleged infringer (and the patentee), or may not. The size of the slipstreaming effect depends on the nature of rivalry.

We demonstrate this with a simple model of imperfect competition. Suppose n firms compete imperfectly as oligopolists. Initially we will consider just one period of competition. Let inverse demand be $P = A - \sum_i q_i$. Production carries a constant marginal cost per unit. Of the n firms, two are *leading* firms that innovate and generally have superior technology while $n-2$ *trailing* firms have access only to publicly available technology. Initially the public technology permits production at cost $\overline{\kappa}$. One of the leading firms already has access to a technology that permits production at cost $\underline{\kappa} < \overline{\kappa}$, while the second of these firms is attempting to innovate its technology to produce below $\underline{\kappa}$. Let $\underline{\kappa}$ be such that, relative to the public technology, the improvement is *drastic* in the sense of Arrow (1962)—if just one firm owns the technology, then the monopoly price $p_m < \overline{\kappa}$. In that case, the monopolist firm earns profit $\pi_M = \frac{(A-\underline{\kappa})^2}{4}$.

Assume that it is optimal for the second firm to spend resources to attempt to innovate. And when the second firm succeeds in innovating, it either innovates to produce with technology $\underline{\underline{\kappa}} < \underline{\kappa}$ (where $\underline{\underline{\kappa}}$ is drastic with respect to $\underline{\kappa}$) or it innovates to some exogenous intermediate point $\underline{\kappa}' \in \left(\underline{\underline{\kappa}}, \underline{\kappa}\right)$. Upon an innovation, the incumbent firm learns from its rival's technology and modifies its own processes to achieve the same (lower) marginal cost. The entrant sues to enforce its patent and block the incumbent firm's imitation. The outcome of litigation precedes market competition. If the innovator's technology achieves $\underline{\underline{\kappa}}$, its patent will be found "valid and infringed" and the incumbent is left to produce at cost $\underline{\kappa}$. Otherwise the innovator loses the case and it may not exclude its rival from technology $\underline{\kappa}'$. If the patent is found "not infringed," then the two firms essentially compete as duopolists, because none

[8]In the settlement agreement, Barr was to revert to a Paragraph IV certification if Zeneca's patent were to be found invalid (Carrier 2009). This would have helped delay subsequent generic entry but most likely would not have prevented it.

of the trailing firms may use either of the leading firms' technologies.[9] If the patent is found "invalid," however, then the trailing firms and the incumbent alleged infringer may use the $\underline{\kappa}'$ technology (which is now publicly available) and there is n-firm oligopoly competition.

The nature of competition is important for profitability. Under Bertrand competition, when the alleged infringer wins the case, at least two firms can produce at cost $\underline{\kappa}'$. Hence, oligopoly competition always yields equilibrium prices equal to $\underline{\kappa}'$ and zero profits for all producing firms. In this case, profits for both of the leading firms are the same—zero—under non-infringement and invalidity. There is no slipstreaming effect.

Under Cournot competition, when the alleged infringer wins the case, firms earn positive profits that depend upon the decision. Under non-infringement, each of the two leading firms earns profit of $\frac{(A-\underline{\kappa}')^2}{9}$. Under invalidity, all n firms produce at cost $\underline{\kappa}'$ and each firm earns a profit of $\left(\frac{A-\underline{\kappa}'}{n+1}\right)^2$. As the profit under non-infringement is higher, there is a strictly positive slipstreaming effect.

For both types of competition, both leading firms earn the same payoffs. If one period of competition is their only consideration, they weakly prefer non-infringement to invalidity and gain the same amount from non-infringement. Under Cournot competition, they strictly prefer non-infringement to invalidity.

2.2. Blocking

Innovative firms often base decisions upon the benefits they expect to receive in multiple periods of competition. In choosing or developing technology, they also factor in whether other firms own valid patents that cover closely related technology. If such related patents are invalidated, then the covered technology becomes freely available. For current market competition, rivals may simply adopt technology for their own use (as in the previous subsection).

In the future, firms may respond to invalidation of other firms' patents by increasing cumulative innovation using the (formerly covered) technology. Indeed, Galasso and Schankerman (2015) show that for patents invalidated by the US Court of Appeals for the Federal

[9] Given our assumption that the $\underline{\kappa}$ technology is drastic relative to the public technology, the price under duopoly competition is always below $\overline{\kappa}$, so fringe firms cannot profitably produce.

Circuit during 1983-2008, forward citations increase by about 50% in the five years after invalidation. They interpret this as reflecting an unblocking effect of patent invalidity.

Returning to our simple model, suppose that there is a second period of competition. Each of the two leading firms would like to start the second period with the dominant ($\underline{\underline{\kappa}}$) technology, but if the first-period entrant's patent is not found "valid and infringed," it is ambiguous which firm will attain this position. Suppose that both leading firms scramble to develop technology $\underline{\underline{\kappa}}$ and just one will succeed. Suppose further that the entrant wins the scramble and replaces the incumbent with probability .5 under patent invalidity, and with probability $.5 + \Delta\rho$ under non-infringement. Intuitively, the first-period incumbent finds successful innovation easier when it (cumulatively) uses the first-period entrant's technology. The blocking effect of patent validity is then the increase in the probability that the entrant patentee wins the scramble, $\Delta\rho$, when non-infringement replaces invalidity.

Just factoring in the dynamic incentives, the firms may have opposing preferences regarding the outcome of first-period patent litigation. With a strictly positive blocking effect, the patentee strictly prefers non-infringement but the alleged infringer strictly prefers invalidity. Factoring in both current-period and future incentives, we see that the patentee always prefers non-infringement to invalidity, while the alleged infringer's preferences depend upon the relative sizes of the slipstreaming and blocking effects.

In the next section, we introduce dynamics. To keep the interpretation of the current-period payoffs as simple as possible, we will assume that an innovation is either drastic with respect to its main rival or allows the innovator to exactly reach parity with the incumbent firm.[10] This simplification permits a straightforward way to specify current-period benefits of patent non-infringement (and the slipstreaming effect) in reduced form. Even given this simplification, we can still specify the two firms' payoffs flexibly to possibly capture asymmetries driven by technology incumbency. Indeed, our specification will also capture cases where innovations affect demand, which are emphasized more heavily in the growth literature (e.g. Grossman and Helpman (1991), Aghion and Howitt (1992), and Aghion et

[10]For the model in section 2, this would mean $\underline{\kappa}' \to \underline{\kappa}$. Note that this assumption yields the additional benefit that it is not necessary to specify whether the incumbent modifies its technology to match the entrant's cost upon non-infringement, or simply uses its own technology unmodified (and produces at a different cost).

al. (1992)) and by Segal and Whinston (2007).[11] We will model the blocking effect in the same way as in this subsection.

3. A Dynamic Model

Consider the following adaptation of the basic model of Segal and Whinston (2007). Let there be an infinite number of discrete periods. Two firms discount future payoffs at rate $\delta \in (0,1)$. In each period, one firm is the *incumbent* (I) while the other is the *entrant* (E). At the beginning of a period, the incumbent is one rung higher than the entrant on the quality ladder. The entrant chooses a rate of *R&D* ϕ. We will interpret this as the probability that the entrant's innovation is strong enough for it to advance at least one rung and obtain a patent. The cost of *R&D* is $c(\phi)$, where $c(\cdot)$ is twice continuously differentiable, increasing and convex. We also assume $\lim_{\phi \to \frac{1}{2}} c'(\phi) = \infty$, which we will call the *Incumbency Advantage* condition. When the incumbency advantage condition holds the equilibrium $\phi < \frac{1}{2}$, which guarantees that the incumbent expects a higher present-discounted payoff than the entrant.[12] If the entrant fails to innovate, then the incumbent earns current-period payoff π_M, the entrant earns current-period payoff 0, and both firms remain in their positions until the beginning of the next period.

If the entrant is successful in innovating, then absent patent litigation the incumbent will costlessly imitate the entrant. Hence, the entrant sues to enforce its patent,[13] and the ensuing litigation costlessly resolves uncertainty about the success of the entrant's innovation and the strength of its patent.[14] If the entrant innovates and the innovation is sufficient so

[11]The results for the limiting case $\underline{\kappa}' \to \underline{\kappa}$ are isomorphic to a specification where firms' innovations increase the size of parameter A drastically, the incumbent has achieved innovation $\overline{A}$, and an entrant that partially succeeds at innovation achieves innovation $\overline{A'}$, with $\overline{A'} \to \overline{A}$.

[12]This is a natural assumption, given that we wish to focus on cases where firms compete "for the market." An incumbency advantage means merely that firms prefer to "win" competition for the market. Footnote 26 shows how this assumption guarantees a positive incumbency advantage, and discusses how a negative incumbency advantage could emerge.

[13]Willingness to sue by the entrant is guaranteed by the promise of a higher payoff as incumbent, but if the incumbent were to preemptively challenge the patent this would not affect any behavior (in choosing legal regime) in litigation. Implicitly, unsuccessful innovation includes all cases where the entrant's innovation is patentable but is so weak that the entrant is uncompetitive and has no chance to replace the incumbent; in that case neither firm is willing to sue.

[14]It is without loss of generality to assume that all current-period payoffs (conditional on innovation) are net of a positive constant litigation cost.

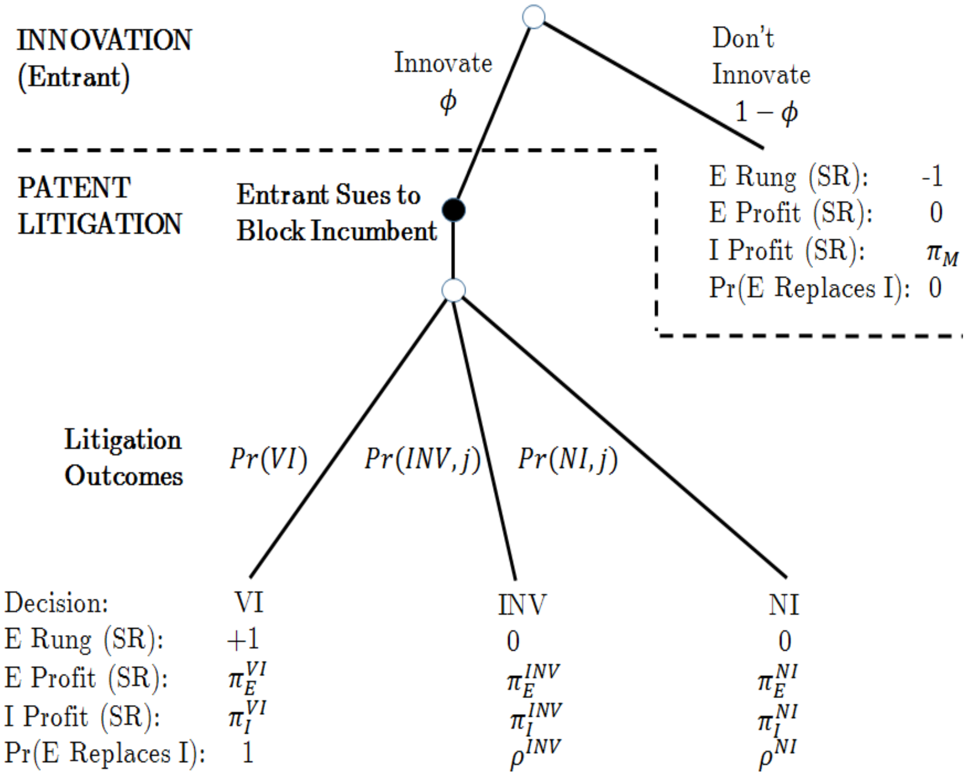

Figure 1: *Innovation and Patent Litigation*

Note: This Figure shows the different outcomes conditional on a choice ϕ of innovation intensity by the entrant, and on legal regime $j \in \{IF, VF\}$. The short run is denoted SR and pertains to current-period outcomes. Pr(I Changes) is the probability that the entrant is the incumbent at the beginning of the next period.

that the entrant can occupy the rung above the current incumbent (rung +1), then the patent is found valid and infringed.[15] The entrant earns current-period payoff $\pi_E^{VI} \leq \pi_M$, the incumbent earns current-period payoff $\pi_I^{VI} \leq \pi_M$, and the entrant replaces the incumbent for certain in the next period. If the innovation is insufficient for rung +1, then the entrant is able only to reach rung 0. Then, the entrant and incumbent simultaneously compete for current-period payoffs and scramble to achieve the next rung (and incumbency) by the beginning of the next period. Both current-period payoffs and the outcome of the scramble depend upon the determination of patent litigation.

Figure 1 summarizes the competitive environment for different outcomes of innovation and litigation. This environment generalizes the basic model from Segal and Whinston (2007). In particular, the case where the patent is always found "valid and infringed"

[15] Implictly, the incumbent tries to imitate the entrant's innovation but is excluded from doing so.

($Pr(VI) = 1$) recovers their model.[16] The remaining cases of patent invalidity and non-infringement are new.

If the patent is determined invalid (INV), then the entrant's technology is freely available and fringe firms can imitate it. The entrant earns a profit of π_E^{INV} and the incumbent earns π_I^{INV}.[17] The entrant replaces the incumbent in the next period with probability $\rho^{INV} \in [0, 1]$.

If the patent is found not infringed (NI), then the entrant's patent remains valid. Because its technology remains proprietary, other firms cannot imitate it. The incumbent and entrant enjoy short run payoffs $\pi_I^{NI} \geq \pi_I^{INV}$ and $\pi_E^{NI} \geq \pi_E^{INV}$, respectively. The entrant replaces the incumbent in the next period with probability $\rho^{NI} \in [\rho^{INV}, 1]$.

Absent infringement, patent validity confers a current-period benefit ($\pi_i^{NI} \geq \pi_i^{INV}$) to both firms. Motivated by the analysis in section 2, we term the *slipstreaming effect* to be $\pi_I^{NI} - \pi_I^{INV}$, the alleged infringer's extra current-period profit due to non-infringement instead of invalidity. It may be strictly positive (e.g., Cournot) or zero (e.g., Bertrand).

The replacement probability is higher under non-infringement than under invalidity ($\rho^{NI} \geq \rho^{INV}$). This captures a future *blocking effect* of patent validity. Under non-infringement, the incumbent must invent around the entrant's technology in scrambling for the next rung. Under invalidity, the incumbent may freely imitate the entrant's technology for the purposes of cumulative innovation.

Consider next the conditional probabilities of non-infringement and invalidity, $Pr(NI|j)$ and $Pr(INV|j)$ respectively. To capture how variation in the legal regime (j) affects these probabilities, we appeal to a simple version of the patent litigation model of Henry and Turner (2016). Let $\theta_{INF} \in \{0, 1\}$ denote the status of a patent's infringement (with "1" assigned to an infringed patent), and let $\theta_{VAL} \in \{0, 1\}$ denote the status of a patent's validity (with "1" assigned to a valid patent). For the four permutations of these two variables, let the

[16] To be complete, there are some additional cosmetic notational changes, i.e., change π_I^{VI} to π_I and change π_E^{VI} to π_E.

[17] Implicitly, we assume that the current-period payoffs include costs (treated as exogenous) of scrambling.

probability distribution be

Permutation	Probability
$\theta_{INF} = 1, \theta_{VAL} = 1$	p_{11}
$\theta_{INF} = 1, \theta_{VAL} = 0$	p_{10}
$\theta_{INF} = 0, \theta_{VAL} = 1$	p_{01}
$\theta_{INF} = 0, \theta_{VAL} = 0$	p_{00},

with $p_{11} + p_{10} + p_{01} + p_{00} = 1$.[18] For the first permutation, the patentee wins on both validity and infringement. Both inquiries are decisive, so the determination is "valid and infringed" (VI). For the second and third permutations, the patentee wins one inquiry, loses the other, and therefore loses the case. The determinations, "invalid" (INV) and "not infringed" (NI) respectively, reflect the decisive (losing) inquiries.[19] For the fourth permutation, the patentee would lose on both inquiries. Thus, neither inquiry is necessarily decisive and the court determination is not obvious.

We consider two legal regimes from the model of Henry and Turner (2016). Under the "Infringement First" (IF) regime, infringement is always considered but validity is considered only if the patent is found to be infringed. Patents that are both invalid and not infringed are always determined "not infringed," i.e., $Pr\,(VI|IF) = p_{11}$, $Pr\,(INV|IF) = p_{10}$ and $Pr\,(NI|IF) = p_{01} + p_{00}$. Under the "Validity First" (VF) regime, validity is always considered but infringement is considered only if the patent is found to be valid. Patents that are both invalid and not infringed are always determined "invalid," i.e., $Pr\,(VI|VF) = p_{11}$, $Pr\,(INV|VF) = p_{10} + p_{00}$ and $Pr\,(NI|VF) = p_{01}$.[20] For the rest of this section, we treat the regime $j \in \{VF, IF\}$ as exogenous.[21]

[18]In the full Henry and Turner (2016) model, θ_{INF} and θ_{VAL} are continuously distributed on a $[0,1] \times [0,1]$ support, the court adopts *decision standards* $\overline{\theta}_{INF}$ and $\overline{\theta}_{VAL}$, and validity and infringement are determined based on whether θ_{INF} and θ_{VAL} are higher or lower than the (respective) decision standards. Here, we implicity assume that $\overline{\theta}_{INF} \in (0,1)$ and $\overline{\theta}_{VAL} \in (0,1)$ are exogenous.

[19]For example, in the second permutation, switching θ_{INF} from 1 to 0 has no effect on whether the patentee wins, so infringement is not decisive. Hence, the determination "invalid" reflects the fact that validity is decisive.

[20]Note that the probability that the patentee wins the case is not affected by the legal regime. We therefore write $Pr\,(VI|VF) = Pr\,(VI|IF) \equiv Pr(VI)$.

[21]The Henry and Turner (2016) model defined these regimes the same way but referred to them as Infringement Senior and Validity Senior. To avoid confusion with the "innovation supply" curve, we change the names of these regimes to Infringement First and Validity First so we can use IF and VF as abbreviations. The Henry and Turner (2016) model also includes a parameter reflecting the probability that Infringement First obtains. We will see later that this parameter may emerge in equilibrium when the regime is endogenous.

Letting $\mathbb{E}^j[\cdot]$ denote the expectation operator for the probabilities associated with regime j, the expected short run payoffs for firm $i \in \{E, I\}$ under the two regimes are

$$\begin{aligned}\mathbb{E}^{IF}[\pi_i] &\equiv Pr(VI)\pi_i^{VI} + Pr(NI|IF)\pi_i^{NI} + Pr(INV|IF)\pi_i^{INV} \\ \mathbb{E}^{VF}[\pi_i] &\equiv Pr(VI)\pi_i^{VI} + Pr(NI|VF)\pi_i^{NI} + Pr(INV|VF)\pi_i^{INV}.\end{aligned}$$

We can write

$$\mathbb{E}^{IF}[\pi_i] - \mathbb{E}^{VF}[\pi_i] = p_{00}\left(\pi_i^{NI} - \pi_i^{INV}\right) \geq 0.$$

Under IF, the probability of a "not infringed" decision is higher and the probability of an "invalid" decision is lower. Hence, each firm earns a higher expected current-period payoff under IF. For the incumbent, this is due to the slipstreaming effect.

Expected replacement probabilities are:

$$\begin{aligned}\mathbb{E}^{IF}[\rho] &= Pr(VI) + Pr(NI|IF)\rho^{NI} + Pr(INV|IF)\rho^{INV} \\ \mathbb{E}^{VF}[\rho] &= Pr(VI) + Pr(NI|VF)\rho^{NI} + Pr(INV|VF)\rho^{INV}.\end{aligned}$$

We can write

$$\mathbb{E}^{IF}[\rho] - \mathbb{E}^{VF}[\rho] = p_{00}(\rho^{NI} - \rho^{INV}) \geq 0.$$

Hence, the expected replacement probability is higher under IF. The incumbent expects a higher probability of losing incumbency under IF, due to the future blocking effect of patent validity.

Like Segal and Whinston (2007), we use dynamic programming to solve for stationary Markov perfect equilibria of an infinite-horizon game. Let V_i^j denote the expected present discounted value, at the beginning of a period, of being firm $i \in \{E, I\}$ under regime $j \in \{IF, VF\}$. Then given the probabilities of innovation and the outcomes of the patent cases, the values for the incumbent and entrant under regime j are

$$\begin{aligned}V_I^j &= \pi_M + \delta V_I^j \\ &\quad + \phi\left[\mathbb{E}^j[\pi_I] - \pi_M + \delta \mathbb{E}^j[\rho](V_E^j - V_I^j)\right] \\ V_E^j &= \delta V_E^j \\ &\quad + \phi\left[\mathbb{E}^j[\pi_E] + \delta \mathbb{E}^j[\rho](V_I^j - V_E^j)\right] - c(\phi).\end{aligned} \tag{1}$$

A potential entrant's level of innovation is chosen to maximize its expected discounted value. Let

$$w \equiv \mathbb{E}^j[\pi_E] + \delta \mathbb{E}^j[\rho](V_I^j - V_E^j) \tag{2}$$

denote the innovation prize where, following Gans (2017), we refer to $V_I^j - V_E^j$ as the *incumbency advantage* under future application of legal regime j. Note that the optimal level of innovation conditional on w is

$$\Phi(w) = \arg\max_{\phi \in [0,1]} \{\phi w - c(\phi)\}.$$

Given our assumptions on $c(\phi)$, $\Phi(w)$ is the unique solution to the entrant's first-order condition $w = c'(\phi)$. This yields the "Innovation Supply" (IS) curve.

Consider now what determines the innovation prize w. Solving for the incumbency advantage and substituting into w, we can write

$$W^j(\phi) = \mathbb{E}^j[\pi_E] + \delta \mathbb{E}^j[\rho] \left(\frac{\phi \left[\mathbb{E}^j[\pi_I] - \mathbb{E}^j[\pi_E]\right] + (1-\phi)\pi_M + c(\phi)}{1 - \delta + 2\delta\phi \mathbb{E}^j[\rho]} \right). \tag{3}$$

It describes the "Innovation Benefit" (IB), which is the value of being the innovator conditional on the rate of innovation. We assume that the IB and IS curves intersect just once, so there is a unique (ϕ^*, w^*). Existence follows from continuity of $\Phi(\cdot)$ and $W^j(\cdot)$, and uniqueness obtains whenever $\pi_M \geq \mathbb{E}^j[\pi_I] + \mathbb{E}^j[\pi_E]$, so that industry profits with a monopoly are higher than expected industry profits are when innovation occurs.

The IS curve does not depend upon the legal regime. Hence, if a change in legal regime shifts the IB curve up for all ϕ, then equilibrium innovation increases. This may shift due to changes in current-period or dynamic payoffs, and the legal regime affects both. First, the innovation benefit $W^j(\phi)$ is increasing in expected short run payoffs $\mathbb{E}^j[\pi_E]$ and $\mathbb{E}^j[\pi_I]$. To see this, rewrite (3) as

$$W^j(\phi) = \frac{\left[1 - \delta(1 - \phi \mathbb{E}^j[\rho])\right] \mathbb{E}^j[\pi_E] + \delta \mathbb{E}^j[\rho] \left[\phi \mathbb{E}^j[\pi_I] + (1-\phi)\pi_M + c(\phi)\right]}{1 - \delta + 2\delta\phi \mathbb{E}^j[\rho]}.$$

This implies that the innovation benefit is higher under IF whenever patent validity confers current-period benefits to both firms.

Second, the innovation benefit $W^j(\phi)$ is increasing in the expected replacement probability. To see this, note that the numerator of the second term in (3) is positive due to the incumbency advantage assumption, and that the entire term is proportional to $\frac{\delta \mathbb{E}^j[\rho]}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]}$, the expected discounted term of incumbency. If the replacement probability is strictly higher under non-infringement, then $\mathbb{E}^{IF}[\rho] > \mathbb{E}^{VF}[\rho]$ and $W^j(\phi)$ is higher under IF.[22]

Hence, the IF regime yields a higher innovation benefit and a higher equilibrium level of innovation. We then have the following result.

Proposition 1. *The equilibrium rate of innovation under IF, ϕ^{IF}, is weakly higher than the equilibrium rate of innovation under VF, ϕ^{VF}. If non-infringement confers a strict current-period benefit for either firm or a strictly higher replacement probability, then equilibrium innovation is strictly higher under IF.*

Figure 2 illustrates the comparison of equilibrium innovation levels ϕ^{IF} and ϕ^{VF} under the IF and VF regimes, respectively. With a strictly positive blocking effect ($\rho^{NI} > \rho^{INV}$), innovation incentives are strictly higher under IF under weak current-period benefits (e.g., Bertrand) and strict current-period benefits (e.g., Cournot). With no blocking effect, innovation incentives are strictly higher under IF whenever either firm enjoys a strict current-period benefit from patent validity.

4. Endogenous Legal Regime

In practice, the sequence of deciding validity and infringement is endogenous to the case. Patentees and alleged infringers may file pre-trial motions for summary judgment, and in some instances juries are specifically instructed to consider validity (infringement) only if the patent is first found to be infringed (valid). To study endogenous sequencing, suppose that after innovation occurs, both the incumbent and entrant may express a preference over whether infringement or validity is determined first. Following court practice, we assume

[22] The careful reader will note that, conditional on *being* the incumbent, the probability of remaining the incumbent is higher under regime j when $\rho^j < \rho^k$. This is reflected in a lower denominator, under regime j, for the term in parentheses. However, this effect is more than offset by the higher probability of *attaining* incumbency under regime k.

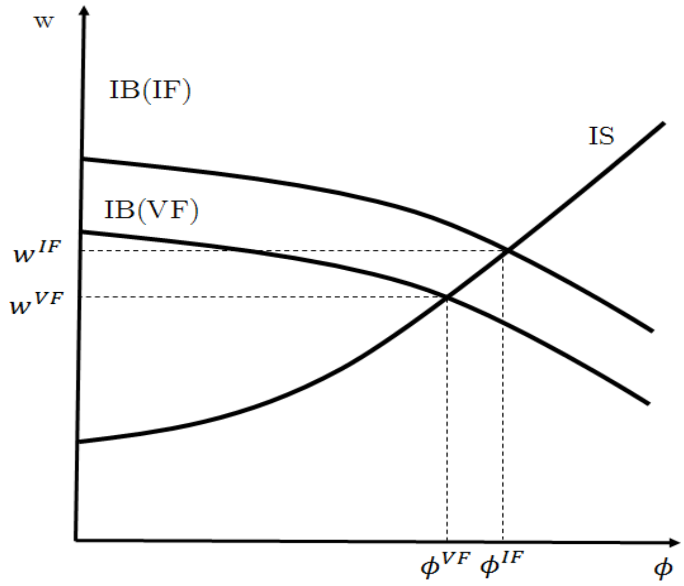

Figure 2: *Equilibrium Innovation,* $\rho^{NI} > \rho^{INV}$

Note: This Figure shows the "Innovation Benefit" (IB) curves for two different legal regimes, along with the "Innovation Supply" (IS) curve. Under the Infringement First (IF) regime, infringement is decided first and validity is considered only after an initial finding that the patent is infringed. Under the Validity First (VF) regime, validity is decided first and infringement is considered only after an initial finding that the patent is valid.

that invalidity moots the infringement question but non-infringement does not moot the validity question. Hence, if both prefer IF, then IF is used. But if either the incumbent or entrant prefers VF, then VF is used.

We solve for equilibrium that is stationary in both the legal regime and the level of innovation. We define an *IF equilibrium* as occurring when both firms optimally choose the IF legal regime, conditional on expecting the IF legal regime and innovation level ϕ^{IF} to be chosen in all future periods. Similarly, we define a *VF equilibrium* as occurring when at least one firm optimally chooses the VF legal regime, conditional on expecting the VF legal regime and innovation level ϕ^{VF} to be chosen in all future periods.

Let the incumbency advantage under regime k and conditional on innovation level ϕ be $\Delta V^k(\phi)$. Conditional on successful innovation, the entrant's and incumbent's payoffs under regime j in the present period, given that from the next period forward legal regime k and the associated equilibrium innovation level ϕ^k obtain, are

$$\Pi_E(j,k,\phi^k) = \delta V_E^k + \mathbb{E}^j[\pi_E] + \delta \mathbb{E}^k[\rho]\Delta V^k(\phi^k)$$
$$\Pi_I(j,k,\phi^k) = \delta V_I^k + \mathbb{E}^j[\pi_I] - \delta \mathbb{E}^k[\rho]\Delta V^k(\phi^k)$$

In a stationary equilibrium, the entrant's payoff (net of δV_E^k) when legal regime j is chosen equals $W^j(\phi^j)$, its equilibrium reward to innovation.

Defining $\Delta\Pi_i(k,\phi^k) \equiv \Pi_i(IF,k,\phi^k) - \Pi_i(VF,k,\phi^k)$ to be the *IF incentive* for firm i, we can write

$$\begin{aligned} \Delta\Pi_E(k,\phi^k) &= p_{00}\left[\pi_E^{NI} - \pi_E^{INV} + \delta\left(\rho^{NI} - \rho^{INV}\right)\Delta V^k(\phi^k)\right] \\ \Delta\Pi_I(k,\phi^k) &= p_{00}\left[\pi_I^{NI} - \pi_I^{INV} - \delta\left(\rho^{NI} - \rho^{INV}\right)\Delta V^k(\phi^k)\right]. \end{aligned} \tag{4}$$

If the IF incentive is strictly positive for firm i, then firm i prefers the IF regime.

To simplify the analysis, we assume that the entrant prefers IF in the case where $\Delta\Pi_E(k,\phi^k) = 0$.[23] It is clear that $\Delta\Pi_E(k,\phi^k) \geq 0$, so the entrant always prefers IF.[24] However, the incumbent may strictly prefer either IF or VF. It enjoys higher expected current-period benefits under IF due to the slipstreaming effect, but is more likely to remove the blocking effect and remain the incumbent under VF. The incumbent's preference is therefore decisive.

Proposition 2. *The (incumbent) alleged infringer's preferences over the legal regime are decisive.*

This proposition greatly simplifies the analysis of equilibrium. The equilibrium regime is determined by the incumbent and the incumbent's preference over non-infringement and invalidity are determined by the trade-off between the current-period slipstream effect and the future blocking effect. An IF equilibrium exists only if $\Delta\Pi_I(IF,\phi^{IF}) \geq 0$. If in addition

[23] This allows us to ignore the unimportant case where the entrant is indifferent between choosing IF and VF. In this case, the entrant may arbitrarily choose VF and be decisive. However, this case obtains only if $\pi_E^{NI} = \pi_E^{INV}$ and $\rho^{NI} = \rho^{INV}$, which is uninteresting because the regime choice is irrelevant and the equilibrium innovation is the same under IF or VF.

[24] A positive incumbency advantage guarantees this. For a negative incumbency advantage, it is necessary to have $\mathbb{E}^j[\pi_I] - \mathbb{E}^j[\pi_E] < 0$. The most straightforward case, mathematically, is where $Pr(VI)$ is close to 1, π_E^{VI} is close to π_M, and π_I^{VI} is close to 0. Even with this, $c(\phi)$ must still be such that the equilibrium ϕ is above $\frac{1}{2}$. In this case, the incumbent would prefer to lose incumbency to the entrant, so that it would get to be the (innovative) entrant in the next period. It would then always prefer the IF regime, while the entrant would potentially prefer VF (which improves its chance to remain the entrant).

$\Delta\Pi_I(VF, \phi^{VF}) > 0$, then the IF equilibrium is unique. Similarly, a VF equilibrium exists only if $\Delta\Pi_I(VF, \phi^{VF}) < 0$. If in addition $\Delta\Pi_I(IF, \phi^{IF}) < 0$, then the VF equilibrium is unique.

We start by discussing instances where particular values of primitives yield clear equilibrium outcomes. The simplest case occurs where there is no blocking effect, $\rho^{NI} = \rho^{INV}$. Then the incumbency advantage is irrelevant to the choice of legal regime and the IF incentive is $\Delta\Pi_I(k, \phi^k) = p_{00}\left[\pi_I^{NI} - \pi_I^{INV}\right] \geq 0$ for all k. Since the incumbent's preference over regimes is decisive, the equilibrium is determined by the slipstream effect.

Proposition 3. *Suppose the blocking effect of patent non-infringement is zero. If the slipstreaming effect is strictly positive, then there is a unique IF equilibrium. If there is no slipstreaming effect, then there are multiple equilibria, including VF.*

Intuitively, if an industry has two leading firms that do not compete fiercely on price and have highly differentiated technologies (so that it is not difficult to invent around patents), alleged infringers will tend to prefer not to litigate validity. If there is also no slipstreaming effect, then the incumbent is indifferent between IF and VF and may, decisively, choose either.

Next, consider the case where there is no slipstreaming effect but there is a blocking effect. In this case, current-period benefits of patent non-infringement are zero, and the IF Incentive $\Delta\Pi_I(k, \phi^k) = -p_{00}\left[\delta\left(\rho^{NI} - \rho^{INV}\right)\Delta V^k(\phi^k)\right] \leq 0$. The incumbent prefers to limit the blocking effect by choosing VF, which obtains as a unique equilibrium.

Proposition 4. *Suppose the slipstreaming effect of patent non-infringement is zero. If there is a strictly positive blocking effect, then there is a unique VF equilibrium.*

Intuitively, if firms do compete fiercely on price and have homogeneous technologies, alleged infringers will prefer to litigate validity.

Propositions 1, 3, and 4 imply that with regime choice, the maximal rate of innovation may not obtain.

Corollary 1. *If the slipstreaming effect is zero, then the equilibrium rate of innovation is lower with regime choice than under mandated IF.*

When VF is chosen in equilibrium, we say that the *litigation strategy effect* reduces the level of innovation. By reducing the entrant's expected current-period payoff and the probability that the entrant's innovation wins incumbency for it, the incumbent's choice of VF lowers the entrant's incentives to innovate.

With strictly positive slipstreaming and blocking effects, multiple equilibria are possible in this model and this externality may be self-reinforcing. To see this, we start by showing that the incumbency advantage falls with the level of innovation ϕ.

Lemma 1. *Let the legal regime be j and fix the innovation level at ϕ. Then the incumbency advantage $\Delta V^j(\phi)$ is strictly decreasing in ϕ for any $\phi \leq \phi^j$, where ϕ^j is the equilibrium level of innovation conditional on regime j.*

Intuitively, as the rate of R&D and the probability of an innovation rises, the incumbent's likelihood of earning the monopoly payoff and the likelihood of remaining the incumbent fall. This leads to a decrease in the value of being the incumbent, V_I^j. Simultaneously, the likelihood of the entrant earning any short run payoff and the likelihood of becoming the incumbent is rising, yielding an increase in the value of being the entrant, V_E^j.

Now, consider the possibility of multiple equilibria. Mathematically, a necessary condition is $\Delta\Pi_I(IF) \geq 0 > \Delta\Pi_I(VF)$. In such circumstances, an alleged infringer who expects IF to be chosen in the future expects a high rate of innovation in the future and a low incumbency advantage, and so prefers IF now. At the same time, an alleged infringer who expects VF to be chosen in the future expects a low rate of innovation in the future and a high incumbency advantage, and so prefers VF now.[25] Hence, the presence or absence of the litigation strategy effect may depend on which of multiple equilibria obtain.[26]

[25]There would then also be a mixed-strategy equilibrium σ, where IF is chosen with probability σ, VF is chosen with probability $1 - \sigma$, and σ is such that $\Delta\Pi_I(\sigma) = 0$. See the appendix for a derivation. This strategy is unstable when $\Delta\Pi_I(IF) > 0 > \Delta\Pi_I(VF)$.

[26] One intuitive assumption guarantees that $\Delta\Pi_I(IF) \geq \Delta\Pi_I(VF)$. In particular, we say that the *Symmetric Case* holds when $\pi_I^{NI} - \pi_I^{INV} = \pi_E^{NI} - \pi_E^{INV} \geq 0$, so that non-infringement has the same level effect on the entrant's and incumbent's current-period payoffs. Notably, both the Bertrand and Cournot models sat-

When $\rho^{NI} > \rho^{INV}$ and $\pi_I^{NI} > \pi_I^{INV}$, a taxonomy of equilibrium cases may emerge. Ideally, we would fully characterize equilibrium as a function of primitives and show when IF or VF emerges in equilibrium. Unfortunately, this is not feasible.[27] The following three short subsections show comparative static effects of certain parameters on equilibrium innovation and on the likelihood that an IF equilibrium exists.

4.1. Comparative Statics With Respect to Conditional Current-Period Payoffs

From the discussion of Proposition 1, we know that any increase in the total expected current-period payoffs increases the reward to innovation and therefore raises innovation incentives. From Proposition 2, we know that the entrant's preferences over the legal regime are irrelevant. From (3), it is easily seen that any increase in one or more of the entrant's conditional current-period payoffs reduces the incumbency advantage. Hence, any such increase both increases equilibrium innovation (conditional on regime j) and attenuates the litigation strategy effect.

Proposition 5. *Any increase in a conditional current-period payoff for the entrant, π_E^o for $o \in \{INV, NI, VI\}$, increases innovation incentives and attenuates the litigation strategy effect.*

Intuitively, increases in the entrant's expected payoffs raise the entrant's reward to innovation and reduce the incumbency advantage. Hence, innovation is higher and the firms are more inclined to live and let live.

The incumbent's conditional current-period payoffs have slightly more ambiguous effects. Increases in these parameters all increase equilibrium innovation (conditional on regime j), but have different effects on the IF incentive. Increases in π_I^{INV} and π_I^{VI} both lower the IF

isfy the Symmetric Case. Then given that the difference in expected current-period payoffs ($\mathbb{E}^j[\pi_I] - \mathbb{E}^j[\pi_E]$) is the same for both regimes, it follows that the equilibrium incumbency advantage under the VF regime is strictly higher than the equilibrium incumbency advantage under the IF regime, and $\Delta\Pi_I(IF) > \Delta\Pi_I(VF)$.

[27] In addition to the cases of unique and multiple equilibria discussed above, if ϕ^{IF} and ϕ^{VF} are such that $\Delta\Pi_I(VF) > 0 > \Delta\Pi_I(IF)$, then the only equilibrium involves the incumbent mixing between IF and VF. This case requires somewhat unusual assumptions about current-period payoffs. See the appendix, but note that we can eliminate this case by focusing attention on the Symmetric Case.

incentive and exacerbate the litigation strategy effect. But an increase in π_I^{NI} raises the IF incentive, attenuating the litigation strategy effect.

4.2. Comparative Statics With Respect to Probabilities of Court Outcomes

Holding ϕ fixed, an increase in $\mathbb{E}^j[\rho]$ reduces the expected term of incumbency, directly reducing the benefit of removing the blocking effect. Additionally, the equilibrium ϕ^j (conditional on regime j) is an increasing function of $\mathbb{E}^j[\rho]$ (recall the discussion of Proposition 1), which further reduces the incumbency advantage as shown in Lemma 1. Hence, any change in $\mathbb{E}^j[\rho]$ not based on a change in the blocking effect $\rho^{NI} - \rho^{INV}$ necessarily raises $\Delta\Pi_I(k, \phi^k)$, increasing the likelihood of the existence of an IF equilibrium, reducing the likelihood of a VF equilibrium, and attenuating the litigation strategy effect. The following result discusses changes in the distribution of court outcomes.

Proposition 6. *A change in the distribution of court outcomes that raises the expected replacement probability under any legal regime, increases the IF incentive for all* $k \in \{IF, VF\}$*, making it more likely that an IF equilibrium exists and less likely that a VF equilibrium exists.*

For example, if the distribution of court determinations changes to place more weight on "valid and infringed" and less weight on all other outcomes, then IF is more likely.

4.3. Comparative Statics With Respect to Replacement Probabilities

Changes to replacement probabilities have more nuanced effects on the chance of a particular equilibrium. Increases in ρ^{INV} and ρ^{NI} increase equilibrium innovation, which lowers the incumbency advantage, $\Delta V^j(\phi)$. However, an increase in ρ^{NI} raises the blocking effect, $\rho^{NI} - \rho^{INV}$, which raises the benefit to the incumbent of VF. In contrast, an increase in ρ^{INV} lowers the blocking effect. Hence, we have the following.

Proposition 7. *An increase in the replacement probability under invalidity increases the IF incentive for all* $k \in \{IF, VF\}$*, making it weakly more likely that an IF equilibrium exists* *and weakly less likely that a VF equilibrium exists. A decrease in the replacement probability under invalidity decreases the IF incentive for all $k \in \{IF, VF\}$, making it less likely that an IF equilibrium exists and more likely that a VF equilibrium exists.*

Intuitively, when ρ^{INV} increases, the entrant's chance of winning the scramble after patent invalidation rises. This increases the equilibrium level of innovation, which reduces the incumbency advantage. This also reduces the blocking effect of validity, which reduces the incentive for the incumbent to choose VF. Both changes attenuate the litigation strategy effect.

Comparative statics on ρ^{NI} are ambiguous. While an increase in ρ^{NI} also lowers the incumbency advantage by increasing equilibrium innovation, it increases the blocking effect $\rho^{NI} - \rho^{INV}$ and the incentive for the incumbent to pursue invalidity. The effect on the incumbent's IF incentive is ambiguous for both $k \in \{IF, VF\}$. If there is an increase in replacement probabilities that leaves the blocking effect unchanged, i.e. ρ^{NI} and ρ^{INV} increase by the same amount, then the likelihood that an IF equilibrium obtains increases and the likelihood that a VF equilibrium obtains decreases.

5. Antitrust Policy

In this section, we consider the effects of antitrust policy on litigation strategy and incentives to innovate. Ideally, antitrust policy takes action that improves innovation incentives and (in our model) attenuates the litigation strategy effect. Antitrust policy addresses many types of behavior, including unilateral action by dominant incumbents and multilateral agreements among competitors. We consider both types of policies.

As Segal and Whinston (2007) discuss, there is a "fundamental tension" in how antitrust policy towards unilateral actions affects innovation. Antitrust policy that protects entrants from entry-deterring actions by incumbents raise the payoff that an entrant innovator receives, possibly encouraging innovation. But of course, entrant innovators also hope to become dominant incumbents, in which case they would wish to then deter entry and maintain incumbency.

Among their more interesting results, Segal and Whinston (2007) show cases where this tension is absent—i.e., antitrust policy can unambiguously increase both incumbent and entrant profits, and thereby increase innovation incentives. We now show that this may also emerge in our framework, and then show that antitrust policy may offer the additional benefit of attenuating the litigation strategy effect.

Consider antitrust policy α, where a higher value represents a policy that is more protective of entrants that innovate beyond incumbents. Denote firm $i's$ payoff under litigation outcome $o \in \{INV, NI, VI\}$ as $\pi_i^o(\alpha)$. Suppose that firm $i's$ current-period profit is the sum of $\pi_i^{VI}(\alpha)$ and a constant term that may differ across court outcomes but is not affected by α. Finally, assume $\frac{d\pi_E^{VI}(\alpha)}{d\alpha} > 0$, so that a higher α raises the entrant's payoff under all litigation outcomes. Suppose further that antitrust policy does not affect the monopoly payoff. This captures scenarios where incumbents can take pre-market action that are anticompetitive (e.g., long-term contracting) but firms cannot tailor additional action to the outcome of patent litigation.

Adding our new notation into the innovation benefit (3) and differentiating with respect to α, we find that the derivative is proportional to

$$\left[\delta\phi\mathbb{E}^j[\rho] + (1-\delta)\right]\frac{d\pi_E^{VI}(\alpha)}{d\alpha} + \delta\phi\mathbb{E}^j[\rho]\frac{d\pi_I^{VI}(\alpha)}{d\alpha}$$

Using a very similar expression, Segal and Whinston (2007, p. 1707, equation 2) show that if a strictly more protective antitrust policy does not lower the total current-period profit under a "valid and infringed" decision, then innovation incentives rise. Intuitively, the benefits to entrants are front-loaded. As long as the total expected current-period payoff to the entrant and incumbent does not fall with antitrust enforcement, $\frac{d\pi_E^{VI}(\alpha)}{d\alpha} \geq \frac{-d\pi_I^{VI}(\alpha)}{d\alpha}$, the reward to innovation is higher. Given the assumption that $\frac{d\pi_E^{VI}(\alpha)}{d\alpha} \geq 0$, it follows that any antitrust policy that raises the incumbent's current-period payoff necessarily increases innovation incentives.

Many antitrust policies that raise total current-period payoffs will also reduce the incumbency advantage. Indeed, any such policy that reduces the incumbent's current-period payoff, or raises it by less than the increase to the entrant's payoff, will both *directly* lower the incumbency advantage and (by Lemma 1) indirectly lower it by raising the equilibrium level of innovation. This attenuates the litigation strategy effect.

Proposition 8. *For any strictly more protective antitrust policy that has a bigger absolute effect on the total current-period payoff of the entrant,* $\frac{d\pi_E^{VI}(\alpha)}{d\alpha} \geq |\frac{d\pi_I^{VI}(\alpha)}{d\alpha}|$, *equilibrium innovation and the incumbent's IF incentive increases.*

For example, if an antitrust policy targets predatory pricing, the policy could increase the incumbent's current-period payoff. As long as it does not increase it more than it increases the entrant's payoff, the incumbency advantage falls and innovation incentives rise. Hence, alleged infringers are more inclined to choose IF.

Next, we consider antitrust policy towards multilateral behavior. Results from previous sections suggest that innovation is reduced by agreements (or industry norms) that reinforce the litigation strategy effect, i.e., that promote the VF regime. With an agreement/norm to choose VF, either VF is a unique equilibrium (and the agreement/norm is unnecessary) or the agreement/norm tips the firms towards a VF equilibrium and away from an IF equilibrium.

That this could be a problem highlights the fundamental tension between antitrust policy and intellectual property. In practice, antitrust authorities might view an agreement among direct rivals to choose the IF regime with suspicion. After all, patents are more likely to remain valid under IF, and an industry norm to use it looks like collusion to maintain the market power that comes from patent protection. Norms to choose IF have the same type of effect as an agreement not to pursue invalidity through inter partes review, or to the practice of negotiating a settlement and jointly requesting a court vacate the invalidity ruling, in that both maneuvers directly reduce the number of invalidated patents. With the type of innovation behavior captured by our model, our results suggest both maneuvers would attenuate the litigation strategy effect and increase the innovation prize. Hence, antitrust authorities should use caution in opposing norms to uphold patent validity.

6. Conclusion

In patent litigation, alleged infringers can win by showing either patent non-infringement or patent invalidity and they are able to alter the probabilities over these outcomes through pretrial maneuvering. In this paper, we highlight a pivotal trade off faced by alleged infringers. On one hand, patent non-infringement prevents other competing firms from using the entrants technology, conferring a current-period benefit by shielding the alleged infringer from additional competition. We refer to this as the slipstreaming effect. On the other hand, patent non-infringement also has a blocking effect on the alleged infringers future innovation by preventing it from using the entrants technology in cumulative innovation.

We analyze the incentives to alter the likelihoods of invalidity and non-infringement and the effects that doing so has on innovation within the framework of an augmented version of the "innovative industries" model of Segal and Whinston (2007). We find that, in general, a higher probability of non-infringement (versus invalidity) improves innovation incentives both by raising the expected current-period payoffs that firms receive and by raising the probability an entrant patentee becomes the next dominant incumbent. However, when the blocking effect is large relative to the slipstreaming effect the alleged infringer may prefer and choose to seek a higher probability of invalidity. When present, this litigation strategy effect lowers the equilibrium rate of innovation.

In this context, we also examine the effects of antitrust policy. Antitrust authorities should exercise care when considering future agreements (or norms) among firms to reduce the probability of patent invalidity, because such agreements may help attenuate the litigation strategy effect and increase the rate of innovation. Additionally, antitrust policy that raises the total current-period payoff to firms may directly raise innovation incentives, as noted by Segal and Whinston (2007). Such policies will often also reduce the incumbency advantage, and when they do this attenuates the litigation strategy effect and raises the level of innovation further.

Many researchers have focused on the issue of patentees enforcing or threatening to enforce "bad patents," i.e., patents that should be found invalid in litigation, and have argued for policies that increase the rate of invalidation. Our analysis highlights a potential unintended consequence of such policies, lower rates of innovation. Our results apply as long as patents emerge, at least to some extent, from innovative activity by direct competitors and are not derived from purely strategic patenting considerations.

Importantly, we specify the entrant to always be the patentee. This choice maintains a tight relationship to the model of Segal and Whinston (2007), but our model does not include situations where an innovating entrant must avoid infringing patents held by the incumbent. In such a model, to capture the trade off between the slipstreaming effect and the blocking effect, one must address the possibility of patent claims by both firms and adopt new assumptions on replacement probabilities. Specifically, it would be necessary to allow for cases where the entrant innovates, successfully defends against a patent lawsuit filed by the incumbent, but might still not replace the incumbent; indeed, to get our key tradeoff in such a setting, the replacement probability would need to depend upon whether the entrant wins on invalidity or non-infringement.[28] We leave this more complicated setting for future work, but we note that it is certainly possible to specify such a model such that the main tradeoffs on legal regime choice that we highlight here are still at play. The biggest difference is that the effects of the legal regime on innovation incentives are more nuanced, because the entrant would be more likely to win incumbency under patent invalidity instead of non-infringement. For example, if the blocking effect is sufficiently large relative to the slipstreaming effect, the alleged infringer would prefer the VF legal regime and this choice may maximize innovation incentives.

One would also need to modify our approach to apply it to cases where litigation is initiated by non-practicing entities (NPEs) because NPEs neither compete in/for the market nor innovate to do so. However, the alleged infringer still faces a trade off between the slipstreaming effect and the blocking effect in such cases. Hence, the alleged infringers ability to influence the likelihood of an invalidity ruling with their defense strategy may, in part, explain why we should not expect litigation to be able to comprehensively sort out invalid patents.

In our modeling, we make several simplifying assumptions for analytical tractability. In addition to the assumptions in Segal and Whinston (2007), we make two notable assumptions particular to our focus on patent litigation. First, tractability of the model requires that, following successful innovation by the entrant, *some* firm achieves the next rung. Whereas

[28]For example, one could set the replacement probability equal to 1 under invalidity and below 1 under non-infringement

Segal and Whinston (2007) assume the entrant always achieves the next rung, we assume the probability the entrant replaces the incumbent depends on the uncertain outcomes of patent litigation and the "scramble. The outcome of patent litigation merely alters the distribution of outcomes for who wins the scramble. However, if, in practice, scrambling behavior has additional future implications for rates of innovation, then our model might produce misleading predictions. For example, if patent invalidation leads to scrambling behavior that produces superior innovations (versus those produced under non-infringement) then the VF regime could yield more successful innovation. In such cases, one would ideally make innovation activity in the scramble endogenous, but the challenge would be to keep the model tractable.

Appendix A: Proofs

Proof of Proposition 1. Simple comparative statics show that $W^j(\phi)$ is increasing in $\mathbb{E}^j[\rho], \mathbb{E}^j[\pi_E]$ and $\mathbb{E}^j[\pi_I]$. Under our assumptions, $\mathbb{E}^{IF}[\rho] \geq \mathbb{E}^{VF}[\rho]$ and $\mathbb{E}^{IF}[\pi_i] \geq \mathbb{E}^{VF}[\pi_i]$ for any i. It follows that $W^{IF}(\phi) \geq W^{VF}(\phi)$ and, if either one of the current-period benefits or the blocking effect is also strict, then $W^{IF}(\phi) > W^{VF}(\phi)$. Hence, the innovation benefit is higher, for all ϕ, under IF, and strictly higher when either one of the current-period benefits or the blocking effect is also strict. Because the innovation supply is the same under both regimes, it follows that equilibrium innovation is higher under regime IF, and strictly higher when either one of the current-period benefits or the blocking effect is also strict. **QED**

Proof of Proposition 2. Since $\pi_E^{NI} \geq \pi_E^{INV}$, $\rho^{NI} \geq \rho^{INV}$ and $\Delta V^k(\phi^k) \geq 0$, $\Delta\Pi_E(k, \phi^k) \geq 0$ for both $k \in \{IF, VF\}$. Additionally, since the entrant prefers IF when $\Delta\Pi_E(k, \phi^k) = 0$ for both $k \in \{IF, VF\}$, the entrant always prefers IF. However, the sign of $\Delta\Pi_I(k)$ is ambiguous for both k. Therefore, the incumbent's preferences over IF and VF are decisive. **QED**

Proof of Proposition 3. Suppose the blocking effect is zero, $\rho^{NI} = \rho^{INV}$, equation (4) for the incumbent becomes

$$\Delta\Pi_I = \Delta\Pi_I(IF, \phi^{IF}) = \Delta\Pi_I(VF, \phi^{VF}) = p_{00}\left[\pi_I^{NI} - \pi_I^{INV}\right].$$

If the slipstreaming effect is strictly positive, $\pi_I^{NI} > \pi_I^{INV}$, then $\Delta\Pi_I > 0$ and the incumbent strictly prefers IF. Since the incumbent's preferences are decisive by Proposition 2, there is a unique IF equilibrium. If there is no slipstreaming effect, $\pi_I^{NI} = \pi_I^{INV}$, then $\Delta\Pi_I = 0$. That is, the incumbent is indifferent between IF and VF and any mixture of IF and VF yield the same payoff. Because the incumbent's preferences are decisive there are an infinite number of equilibria. **QED**

Proof of Proposition 4. Suppose the slipstreaming effect is zero, $\pi_I^{NI} = \pi_I^{INV}$, and a strictly positive blocking effect, $\rho^{NI} > \rho^{INV}$, equation (4) for the incumbent becomes

$$\Delta\Pi_I(k, \phi^k) = p_{00}\left[-\delta(\rho^{NI} - \rho^{INV})\Delta V^k(\phi^k)\right].$$

Since $\Delta V^k(\phi^k) > 0$, $\Delta\Pi_I(j, \phi^j) < 0$ for both $j \in \{IF, VF\}$. Hence, the incumbent strictly prefers VF. Because the incumbent's preferences are decisive, there is a unique VF equilibrium. **QED**

Proof of Lemma 1.

$$\begin{aligned}\frac{d\Delta V^j(\phi)}{d\phi} &= \frac{-\pi_M + \mathbb{E}^j[\pi_I] - \mathbb{E}^j[\pi_E] + c'(\phi)}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]} \\ &\quad - \left(\frac{2\delta\mathbb{E}^j[\rho]}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]}\right)\frac{(1-\phi^j)\pi_M + \phi(\mathbb{E}^j[\pi_I] - \mathbb{E}^j[\pi_E]) + c(\phi)}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]} \\ &= \frac{-\pi_M + \mathbb{E}^j[\pi_I] - \mathbb{E}^j[\pi_E] + c'(\phi)}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]} - \left(\frac{2\delta\mathbb{E}^j[\rho]}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]}\right)\Delta V^j(\phi)\end{aligned}$$

For any level of R&D less than or equal to the equilibrium of R&D, i.e. $\phi \in (0, \phi^j]$,

$$c'(\phi) \leq W^j(\phi) = \mathbb{E}^j[\pi_E] + \delta\mathbb{E}^j[\rho]\Delta V^j(\phi),$$

where the expression holds with equality at ϕ^j. Plugging into the equation above gives,

$$\begin{aligned}\frac{d\Delta V^j(\phi)}{d\phi} &\leq \frac{-\pi_M + \mathbb{E}^j[\pi_I] - \mathbb{E}^j[\pi_E] + \mathbb{E}^j[\pi_E] + \delta\mathbb{E}^j[\rho]\Delta V^j(\phi)}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]} - \left(\frac{2\delta\mathbb{E}^j[\rho]}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]}\right)\Delta V^j(\phi) \\ &= -\frac{\pi_M - \mathbb{E}^j[\pi_I]}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]} - \left(\frac{\delta\mathbb{E}^j[\rho]}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]}\right)\Delta V^j(\phi) \\ &< 0\end{aligned}$$

since $\pi_M > \mathbb{E}^j[\pi_I]$ and $\Delta V^j(\phi) > 0$. **QED**

Proof of Proposition 5. From the Proof of Proposition 1, we know that $W^j(\phi)$ is increasing in $\mathbb{E}^j[\pi_E]$, which rises with an increase in any π_E^o. Hence innovation incentives increase. It is also easily seen from the term in parentheses in equation (3) that the incumbency advantage $V_I^j - V_E^j$ is decreasing in $\mathbb{E}^j[\pi_E]$, which also increases with an increase in any π_E^o. Because the incumbent's IF incentive depends on entrant payoffs only through the incumbency advantage, the overall IF incentive falls with an increase in any π_E^o. Hence, the litigation strategy effect is attenuated. **QED**

Proof of Proposition 6. Suppose a change in the distribution of court outcome increases the expected replacement probability, $\mathbb{E}^j[\rho]$, for both $j \in \{IF, VF\}$. Since the innovation benefit $W^j(\phi)$ is increasing in $\mathbb{E}^j[\rho]$, ϕ^j is increasing in $\mathbb{E}^j[\rho]$ and, by Lemma 1, $\Delta V^j(\phi^j)$ is decreasing in $\mathbb{E}^j[\rho]$. It follows from equation (4) that $\Delta\Pi_I(j, \phi^j)$ is increasing in $\mathbb{E}^j[\rho]$. Since $\Delta\Pi_I(j, \phi^j)$ is rising for both $j \in \{IF, VF\}$, the likelihood that an IF equilibrium exists is increasing and the likelihood that a VF equilibrium exists is decreasing. **QED**

Proof of Proposition 7. $\mathbb{E}^j[\rho]$ is increasing in ρ^{INV}. Since the innovation benefit $W^j(\phi)$ is increasing in $\mathbb{E}^j[\rho]$, and hence ρ^{INV}, ϕ^j is increasing in ρ^{INV}. Thus, $\Delta V^j(\phi^j)$ is decreasing in ρ^{INV} by Lemma 1. Additionally, the blocking effect, $\rho^{NI} - \rho^{INV}$, is decreasing in ρ^{INV}. It follows from equation (4) that $\Delta\Pi_I(j, \phi^j)$ is increasing in ρ^{INV}. Therefore, an increase (decrease) in ρ^{INV} makes it weakly more (less) likely that an IF equilibrium exists and weakly less (more) likely that a VF equilibrium exists. **QED**

Proof of Proposition 8. Suppose $\frac{d\pi_E^{VI}(\alpha)}{d\alpha} \geq |\frac{d\pi_I^{VI}(\alpha)}{d\alpha}|$. Differentiating the innovation benefit in equation (3) with respect to α, we find

$$\frac{\partial W(\phi, \alpha)}{\partial \alpha} = \frac{(1-\delta)}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]} \frac{d\pi_E^{VI}(\alpha)}{d\alpha} + \frac{\delta\phi\mathbb{E}^j[\rho]}{1-\delta+2\delta\phi\mathbb{E}^j[\rho]} \left[\frac{d\pi_E^{VI}(\alpha)}{d\alpha} + \frac{d\pi_I^{VI}(\alpha)}{d\alpha}\right].$$

The first term is positive because stronger antitrust policy is assumed to be more protective of entrants, i.e. $\frac{d\pi_E^{VI}(\alpha)}{d\alpha} > 0$, and the second term is positive because total current-period payoffs are not falling in α, i.e. $\frac{d\pi_E^{VI}(\alpha)}{d\alpha} \geq |\frac{d\pi_I^{VI}(\alpha)}{d\alpha}|$. Thus, the entrant's incentive to innovate is rising in α under either regime $j \in \{IF, VF\}$. Additionally, the entrant's innovation incentive would rise further if the regime changes from VF to IF.

The incumbent's IF incentive is

$$\Delta\Pi_I(k, \phi^k(\alpha), \alpha) = \pi_I^{NI} - \pi_I^{INV} - \delta(\rho^{NI} - \rho^{INV})\Delta V^k(\phi^k(\alpha), \alpha).$$

Because π_i^{NI} and π_i^{INV} are not affected by α, $\Delta\Pi_I(j, \alpha)$ increases whenever

$$\Delta V^k(\phi^k(\alpha), \alpha) = \frac{(1 - \phi^k(\alpha))\pi_M + \phi^k(\alpha)\left(\mathbb{E}^k[\pi_I(\alpha)] - \mathbb{E}^k[\pi_E(\alpha)]\right) + c(\phi^k(\alpha))}{1 - \delta + 2\delta\phi^k(\alpha)\mathbb{E}^k[\rho]}$$

decreases. Since $\mathbb{E}^k[\pi_I(\alpha)]$ is falling in α and $\mathbb{E}^k[\pi_E(\alpha)]$ is rising in α, for any fixed ϕ, $\Delta V^k(\phi, \alpha)$ is decreasing in α. For $\alpha' > \alpha$,

$$\Delta V^k(\phi^k(\alpha), \alpha) > \Delta V^k(\phi^k(\alpha), \alpha') > \Delta V^k(\phi^k(\alpha'), \alpha')$$

Starting from the optimal innovation level at α, the first inequality is because $\Delta V^k(\phi, \alpha)$ is decreasing in α. Because the optimal ϕ^k is rising in α, the second inequality follows from Lemma 1. Therefore, for $\alpha' > \alpha$,

$$\Delta\Pi_I(k, \phi^k(\alpha'), \alpha') > \Delta\Pi_I(k, \phi^k(\alpha), \alpha).$$

That is, the incumbent's IF incentive is increasing in α. Hence the likelihood an IF (VF) equilibrium existing is increasing (decreasing) in α, potentially further increasing the entrant's innovation incentive.

Appendix B: Mixed Strategy Equilibrium

Consider a mixed strategy, where the incumbent chooses IF with probability $\sigma \in [0, 1]$. Denote ϕ^σ as the optimal level of innovation conditional on mixed strategy σ being played (we show how to solve for it later), and let V_i^σ be the valuation for being in position i given that the equilibrium follows σ.

The first thing that must hold is that the profit from choosing IF must be the same as from choosing VF:

$$\pi_I^{NI} - \pi_I^{INV} - \delta\Delta\rho(V_I^\sigma - V_E^\sigma) = 0.$$

We can then easily solve to find

$$V_I^\sigma - V_E^\sigma = \frac{\pi_I^{NI} - \pi_I^{INV}}{\delta\Delta\rho}. \tag{5}$$

Next, we can write

$$V_I^\sigma = \sigma V_I^{IF} + (1-\sigma)V_I^{VF}.$$
$$V_E^\sigma = \sigma V_E^{IF} + (1-\sigma)V_E^{VF}.$$

Substituting from (1), we can then solve to find

$$\begin{aligned} V_I^\sigma - V_E^\sigma &= \delta\left(V_I^\sigma - V_E^\sigma\right) + (1-\phi^\sigma)\,\pi_M + c\left(\phi^\sigma\right) \\ &\quad + \phi^\sigma\left[\sigma\left(\mathbb{E}^{IF}[\pi_I] - \mathbb{E}^{IF}[\pi_E]\right) + (1-\sigma)\left(\mathbb{E}^{VF}[\pi_I] - \mathbb{E}^{VF}[\pi_I]\right)\right] \\ &\quad + \phi^\sigma 2\delta\left[\left(\sigma\mathbb{E}^{IF}[\rho] + (1-\sigma)\mathbb{E}^{VF}[\rho]\right)\left(V_E^\sigma - V_I^\sigma\right)\right]. \end{aligned}$$

This reduces to

$$V_I^\sigma - V_E^\sigma = \frac{(1-\phi^\sigma)\,\pi_M + c\left(\phi^\sigma\right) + \phi^\sigma\left[\mathbb{E}^{VF}[\pi_I] - \mathbb{E}^{VF}[\pi_E] + \sigma p_{00}\left(\left(\pi_I^{NI} - \pi_I^{INV}\right) - \left(\pi_E^{NI} - \pi_E^{INV}\right)\right)\right]}{1-\delta+2\delta\phi^\sigma\left[\mathbb{E}^{VF}[\rho] + \sigma p_{00}\Delta\rho\right]}.$$

Setting this expression equal to (5), we can then solve to find

$$\sigma = \frac{\delta\Delta\rho\left[(1-\phi^\sigma)\,\pi_M + c\left(\phi^\sigma\right) + \phi^\sigma\left[\mathbb{E}^{VF}[\pi_I] - \mathbb{E}^{VF}[\pi_E]\right]\right] - \left(1-\delta+2\delta\mathbb{E}^{VF}[\rho]\right)\left(\pi_I^{NI} - \pi_I^{INV}\right)}{\delta\Delta\rho\phi^\sigma p_{00}\left[\left(\pi_I^{NI} - \pi_I^{INV}\right) + \left(\pi_E^{NI} - \pi_E^{INV}\right)\right]}. \tag{6}$$

Finally, we can write the expected reward to innovation as

$$w(\sigma) = \sigma\mathbb{E}^{IF}[\pi_E] + (1-\sigma)\mathbb{E}^{VF}[\pi_E] + \delta\left(\sigma\mathbb{E}^{IF}[\rho] + (1-\sigma)\mathbb{E}^{VF}[\rho]\right)\left(\frac{\pi_I^{NI} - \pi_I^{INV}}{\delta\Delta\rho}\right).$$

where σ follows (6). Then ϕ^σ solves

$$\Phi(w) = \arg\max_{\phi\in[0,1]}\{\phi w(\sigma) - c(\phi)\}.$$

It is clear that innovation is higher when σ is higher, that is, when IF is more likely to be chosen.

References

Bock, J. 2013. "An Empirical Study of Certain Settlement-Related Motions for Vacatur in Patent Cases," *Indiana Law Journal* 88, 919-979.

Carrier, M. 2009. *Innovation for the 21st Century: Harnessing the Power of Intellectual Property and Antitrust Law* New York, NY: Oxford University Press.

Cremers, K.; Gaessler, F.; Harhoff, D.; Helmers, C.; Lefouili, Y. 2016. "Invalid but infringed? An analysis of the bifurcated patent litigation system" *Journal of Economic Behavior & Organization* 131, 218-242.

Cremers, K.; Ernicke, M.; Gaessler, F.; Harhoff, D.; Helmers, C.; McDonagh, L.; Schliessler, P.; Van Zeebroeck, N. 2017. "Patent litigation in Europe" *European Journal of Law and Economics* 44, 1-44.

Economist, 2010. "Smartphone lawsuits: The great patent battle," The Economist, October 21, 2010.

Evans, D.; Schmalensee, R. 2002. "Some Economic Aspects of Antitrust Analysis in Dynamically Competitive Industries." In *Innovation Policy and the Economy* Volume 2, ed. A. Jaffe, J. Lerner, S. Stern, 1-49. Washington, DC: AEI-Brookings Joint Center for Regulatory Studies.

Farrell, J.; Merges, R. 2004. "Incentives to Challenge and Defend Patents: Why Litigation Won't Reliably Fix Patent office Errors and Why Administrative Patent Review Might Help," *Berkeley Technology Law Journal* 19, 943-70.

Ford, R. 2013. "Patent Invalidity Versus Noninfringement," *Cornell Law Review* 99, 71-128.

Frakes, M.; Wasserman, M. Forthcoming. " Irrational Ignorance at the Patent Office," *Vanderbilt Law Review.*

Galasso, A.; Schankerman, M. 2015. "Patents and Cumulative Innovation: Causal Evidence from the Courts," *Quarterly Journal of Economics* 130, 317-69.

Gans, J. 2017. "Economics of Innovation," in *The Cambridge Handbook of Antitrust, Intel-*

lectual Property and High Tech, ed. R. Blair and D. Sokol, Cambridge, UK: University Printing House.

Gans, J.; Persson, L. 2013. "Entrepreneurial Commercialization Choices and the Interaction Between IPR and Competition Policy," *Industrial and Corporate Change* 22, 131-51.

Henry, M,; Turner, J. 2016. "Across Five Eras: Patent Validity and Infringement Rates in United States Courts, 1929-2006." *Journal of Empirical Legal Studies* 13, 454-86.

Lemley, M.A. 2001. "Rational Ignorance at the Patent Office", *Northwestern University Law Review* 95, 1495-1528.

Murray, F.; Stern, S. 2007. "Do Formal Intellectual Property Rights Hinder the Free Flow of Scientific Knowledge? An Empirical Test of the Anticommons Hypothesis," *Journal of Economic Behavior and Organization* 63, 648-687.

Segal, I.; Whinston, M. 2007. "Antitrust in Innovative Industries," *American Economic Review* 97, 1703-30.

Williams, H. 2013. "Intellectual Property Rights and Innovation: Evidence from the Human Genome," *Journal of Political Economy* 121, 1-27.